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A new look at nuclear supersymmetry through transfer experiments

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Abstract

Nuclear supersymmetry is reviewed and some of its applications and extensions are discussed, together with a proposal for new, more stringent and precise tests to probe the supersymmetry classification, in particular, correlations between nuclei that belong to the same supermultiplet. The combination of these theoretical and experimental studies may play a unifying role in nuclear phenomena.

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1. Introduction

Supersymmetric quantum mechanics (SSQM) arose from the concept of supersymmetry in quantum field theory applied to the simpler case of quantum mechanics [1]. This framework has been very fruitful in studying potential problems in quantum mechanics, not only to understand the connections between analytically solvable problems, but also to discover new solutions.

In this paper we discuss a somewhat different application of the concept of supersymmetric quantum mechanics, proposed more than two decades ago in the field of nuclear structure physics [2] and known as nuclear supersymmetry (n-SUSY). This approach has similarities to SSQM, but some significant differences too. While both frameworks treat bosonic and fermionic systems on an equal footing, in the traditional SSQM approach the Hamiltonian H is factorized in terms of the so-called supercharges (concretely, H is the anticommutator of the supercharges), whereas in n-SUSY the Hamiltonian is more general and is a function of the generators of the graded Lie algebra associated with the supergroup which governs the algebraic structure of the problem. In analogy to the case of SSQM, in n-SUSY the fermionic generators of the graded Lie algebra play the role of supercharges which connect bosonic and fermionic systems. Physically they are associated with one-nucleon transfer operators connecting states in different neighbouring nuclei. In general, however, the supercharges do not commute with the Hamiltonian. As a consequence, the spectra of the systems under study are not identical, as they are in SSQM. In other words, while in SSQM H is a generator of the superalgebra, this is not the case in n-SUSY, where a more complicated structure is used. This is a necessary characteristic, because n-SUSY connects the spectroscopic properties of states in even- and odd-mass nuclei, and we know these properties are quite different.

In the present paper, we first describe the formalism of nuclear supersymmetry. Next we report the first results of an ongoing investigation of one- and two-nucleon transfer reactions in the Pt–Au mass region which is considered to provide the best examples of n-SUSY in nature. We establish new correlations between transfer reactions among different pairs of nuclei as a consequence of n-SUSY which can be tested directly in future experiments. Finally, we discuss future perspectives for nuclear supersymmetry, in particular related to some ideas put forward several years ago to generalize n-SUSY to other (transitional) regions of the nuclear mass table [3], and to special correlations between one- and two-nucleon transfer reactions and β decay.

2. Dynamical supersymmetries in nuclear physics

Dynamical supersymmetries were introduced [2] in nuclear physics in 1980 by Iachello in the context of the interacting boson model (IBM) [4] and its extensions. The spectroscopy of atomic nuclei is characterized by the interplay between collective (bosonic) and single-particle (fermionic) degrees of freedom.

The IBM describes collective excitations in even–even nuclei in terms of a system of interacting monopole and quadrupole bosons with angular momentum $l = 0, 2$. The bosons are associated with the number of correlated valence proton and neutron pairs, and hence the number of bosons N is half the number of valence nucleons. Since it is convenient to express the Hamiltonian and other operators of interest in second quantized form, we introduce creation, $s^\dagger$ and $d^\dagger_{\rm m}$, and annihilation, s and $d_{\rm m}$, operators, which altogether can be denoted by $b^\dagger_i$ and b_i with $i = l, m$ ($l = 0, 2$ and $-l \leqslant m \leqslant l$). The operators $b^\dagger_i$ and b_i satisfy the commutation relations

$$\left[b_i, b^\dagger_j\right] = \delta_{ij} \qquad \left[b^\dagger_i, b^\dagger_j\right] = [b_i, b_j] = 0. \tag{1}$$

The bilinear products

$$B_{ij} = b^\dagger_i b_j \tag{2}$$

generate the algebra of $U(6)$, the unitary group in six dimensions

$$[B_{ij}, B_{kl}] = B_{il}\delta_{jk} - B_{kj}\delta_{il}. \tag{3}$$

The IBM Hamiltonian and other operators of interest are expressed in terms of the generators of $U(6)$. In general, the Hamiltonian has to be diagonalized numerically to obtain the energy eigenvalues and wavefunctions. There exist, however, special situations in which the eigenvalues can be obtained in closed, analytic form. These special solutions provide a framework in which energy spectra and other nuclear properties (such as quadrupole transitions and moments) can be interpreted in a qualitative way. These situations correspond to dynamical symmetries of the Hamiltonian [4]. A dynamical symmetry arises when the Hamiltonian is expressed in terms of Casimir invariants of a chain of subgroups of $G = U(6), G \supset G_1 \supset G_2 \supset \cdots$ only. The eigenstates can then be classified uniquely according to the irreducible representations of G and its subgroups $G_1, G_2, \ldots$. The different representations of $G, G_1, G_2 \cdots$ are split but not admixed by the Hamiltonian. The energy eigenvalues are given by the expectation values of the Casimir operators. In addition, by using standard group theoretical techniques it is possible to obtain analytic expressions for electromagnetic transition rates and quadrupole moments, etc.

The concept of dynamical symmetry has been shown to be a very useful tool in different branches of physics. A well-known example in nuclear physics is the Elliott $SU(3)$ model [5] to describe the properties of light nuclei in the sd shell. Another example is the $SU(3)$ flavour symmetry of Gell-Mann and Ne'eman [6] to classify the baryons and mesons into flavour octets, decouplets and singlets and to describe their masses with the Gell-Mann–Okubo mass formula.

For odd-mass nuclei the IBM has been extended to include single-particle degrees of freedom [7]. The interacting boson–fermion model (IBFM) has as its building blocks a set of N bosons with $l = 0, 2$ and an odd nucleon (either a proton or a neutron) occupying the single-particle orbits with angular momenta $j = j_1, j_2, \ldots$. The components of the fermion angular momenta span the m-dimensional space of the group $U(m)$ with $m = \sum_j (2j+1)$. We introduce, in addition to the boson operators for the collective degrees of freedom, fermion creation $a_i^\dagger$ and annihilation a_i operators for the extra nucleon. The fermion operators satisfy anti-commutation relations

$$\left\{a_i, a_j^\dagger\right\} = \delta_{ij} \qquad \left\{a_i^\dagger, a_j^\dagger\right\} = \{a_i, a_j\} = 0. \tag{4}$$

The bilinear products

$$A_{ij} = a_i^\dagger a_j \tag{5}$$

generate the algebra of $U(m)$, the unitary group in m dimensions

$$[A_{ij}, A_{kl}] = A_{il}\delta_{jk} - A_{kj}\delta_{il}. \tag{6}$$

By construction the fermion operators commute with the boson operators.

$$[B_{ij}, A_{kl}] = 0. \tag{7}$$

The operators B_{ij} and A_{ij} generate the Lie algebra of the symmetry group $G = U^B(6) \otimes U^F(m)$ of the IBFM. The dynamical symmetries that can arise in the IBFM are known under the name of dynamical boson–fermion symmetries for odd-mass nuclei.

Boson–fermion symmetries can further be extended by introducing the concept of supersymmetries [8], in which states in both even–even and odd–even nuclei are treated in a single framework. So far, we have discussed the symmetry properties of a mixed system of boson and fermion degrees of freedom for a fixed number of bosons N and one fermion $M = 1$. The operators B_{ij} and A_{ij} can only change bosons into bosons and fermions into fermions. In addition to B_{ij} and A_{ij}, one can introduce operators that change a boson into a fermion and vice versa, but conserve the total number of bosons and fermions

$$F_{ij} = b_i^\dagger a_j \qquad G_{ij} = a_i^\dagger b_j. \tag{8}$$

The enlarged set of operators B_{ij}, A_{ij}, F_{ij} and G_{ij} forms a closed algebra which consists of both commutation and anticommutation relations

$$\begin{aligned}
&[B_{ij}, B_{kl}] = B_{il}\delta_{jk} - B_{kj}\delta_{il} \qquad [B_{ij}, A_{kl}] = 0 \qquad [B_{ij}, F_{kl}] = F_{il}\delta_{jk} \\
&[B_{ij}, G_{kl}] = -G_{kj}\delta_{il} \qquad [A_{ij}, A_{kl}] = A_{il}\delta_{jk} - A_{kj}\delta_{il} \qquad [A_{ij}, F_{kl}] = -F_{kj}\delta_{il} \\
&[A_{ij}, G_{kl}] = G_{il}\delta_{jk} \qquad \{F_{ij}, F_{kl}\} = 0 \qquad \{F_{ij}, G_{kl}\} = B_{il}\delta_{jk} + A_{kj}\delta_{il} \\
&\{G_{ij}, G_{kl}\} = 0.
\end{aligned} \tag{9}$$

$$
\begin{array}{ccccc}
\text{even-odd} & & & & \text{odd-odd} \\
N_\nu+1, N_\pi, j_\pi & & & & N_\nu, N_\pi, j_\nu, j_\pi \\
 & {}^{195}_{79}\mathrm{Au}_{116} & \leftrightarrow & {}^{196}_{79}\mathrm{Au}_{117} & \\
 & \updownarrow & & \updownarrow & \\
 & {}^{194}_{78}\mathrm{Pt}_{116} & \leftrightarrow & {}^{195}_{78}\mathrm{Pt}_{117} & \\
\text{even-even} & & & & \text{odd-even} \\
N_\nu+1, N_\pi+1 & & & & N_\nu, N_\pi+1, j_\nu
\end{array}
$$

Figure 1. Magic quartet of nuclei.

This algebra can be identified with that of the graded Lie group $G = U(6/m)$. It provides an elegant scheme in which the IBM and IBFM can be unified into a single framework [8]

$$G = U(6/m) \supset U^B(6) \otimes U^F(m). \tag{10}$$

In this supersymmetric framework, even–even and odd-mass nuclei form the members of a supermultiplet which is characterized by $[\mathcal{N}\}$ where $\mathcal{N} = N + M$, i.e. the total number of bosons and fermions. Thus, supersymmetry distinguishes itself from other symmetries in that it includes, in addition to transformations among fermions and bosons, transformations that change a boson into a fermion and vice versa.

The Hamiltonian of n-SUSY is written in terms of the generators of the graded Lie algebra of $U(6/m)$ of equation (9). A dynamical supersymmetry arises when the Hamiltonian is composed of the Casimir operators of a chain of subgroups of $U(6/m)$. Dynamical nuclear supersymmetries correspond to very special forms of the Hamiltonian which may not be applicable to all regions of the nuclear chart, but nevertheless several nuclei in the Os–Ir–Pt–Au region have been found to provide experimental evidence for the approximate occurrence of supersymmetries in nuclei.

3. Dynamical neutron–proton supersymmetry

The mass region $A \sim 190$ has been a rich source of possible empirical evidence for the existence of (super)symmetries in nuclei. The even–even nucleus ^{196}Pt is the standard example of the *SO*(6) dynamical symmetry (DS) of the IBM [9]. The odd-proton nuclei ^{191,193}Ir and ^{193,195}Au were suggested as examples of the Spin(6) DS [2], in which the odd-proton is allowed to occupy the $\pi d_{3/2}$ orbit, whereas the pairs of nuclei ^{192}Os–^{191}Ir, ^{194}Os–^{193}Ir, ^{192}Pt–^{193}Au and ^{194}Pt–^{195}Au have been analysed as examples of a $U(6/4)$ supersymmetry [8]. The odd-neutron nucleus ^{195}Pt, together with ^{194}Pt, was studied in terms of a $U(6/12)$ supersymmetry, in which the odd neutron occupies the $\nu p_{1/2}$, $\nu p_{3/2}$ and $\nu f_{5/2}$ orbits [10]. These ideas were later extended to the case where neutron and proton bosons are distinguished [11], predicting in this way a correlation among quartets of nuclei, consisting of an even–even, an odd-proton, an odd-neutron and an odd–odd nucleus. The best experimental example of such a quartet with $U(6/12)_\nu \otimes U(6/4)_\pi$ supersymmetry is provided by the nuclei ^{194}Pt, ^{195}Au, ^{195}Pt and ^{196}Au which are characterized by $\mathcal{N}\pi = N_\pi + 1 = 2$ and $\mathcal{N}\nu = N_\nu + 1 = 5$, see figure 1.

The supersymmetric classification of nuclear levels in the Pt and Au isotopes has been re-examined by taking advantage of the significant improvements in experimental capabilities developed in the last decade. High resolution transfer experiments with protons and polarized deuterons have led to strong evidence for the existence of supersymmetry (SUSY) in atomic nuclei. The experiments include high-resolution transfer experiments to ^{196}Au at TU/LMU München [12, 13], and in-beam gamma ray and conversion electron spectroscopy following the reactions ^{196}Pt (d, 2n) and ^{196}Pt (p, n) at the cyclotrons of the PSI and Bonn [14]. These studies have achieved an improved classification of states in ^{195}Pt and ^{196}Au which give further support to the original ideas [10, 15, 11] and extend and refine previous experimental work [16–18] in this research area.

As we mentioned before, the Pt and Au nuclei have been described in terms of a dynamical $U(6/12)_\nu \otimes U(6/4)_\pi$ supersymmetry. In this case, the relevant subgroup chain is given by [11]

$$\begin{aligned} U(6/12)_\nu \otimes U(6/4)_\pi &\supset U^{B_\nu}(6) \otimes U^{F_\nu}(12) \otimes U^{B_\pi}(6) \otimes U^{F_\pi}(4) \\ &\supset U^{B}(6) \otimes U^{F_\nu}(6) \otimes U^{F_\nu}(2) \otimes U^{F_\pi}(4) \\ &\supset U^{BF_\nu}(6) \otimes U^{F_\nu}(2) \otimes U^{F_\pi}(4) \\ &\supset SO^{BF_\nu}(6) \otimes U^{F_\nu}(2) \otimes SU^{F_\pi}(4) \\ &\supset \mathrm{Spin}(6) \otimes U^{F_\nu}(2) \\ &\supset \mathrm{Spin}(5) \otimes U^{F_\nu}(2) \\ &\supset \mathrm{Spin}(3) \otimes SU^{F_\nu}(2) \\ &\supset SU(2). \end{aligned} \tag{11}$$

The Hamiltonian is expressed in terms of the Casimir operators as

$$H = \alpha C_{2U^{BF_\nu}(6)} + \beta C_{2SO^{BF_\nu}(6)} + \gamma C_{2\,\mathrm{Spin}(6)} + \delta C_{2\,\mathrm{Spin}(5)} + \epsilon C_{2\,\mathrm{Spin}(3)} + \eta C_{2SU(2)}. \tag{12}$$

The corresponding eigenvalues describe simultaneously the excitation spectra of the quartet of nuclei in figure 1

$$\begin{aligned} E = \alpha[N_1(N_1+5) + N_2(N_2+3) + N_3(N_3+1)] + \beta\big[\Sigma_1(\Sigma_1+4) + \Sigma_2(\Sigma_2+2) + \Sigma_3^2\big] \\ + \gamma\big[\sigma_1(\sigma_1+4) + \sigma_2(\sigma_2+2) + \sigma_3^2\big] + \delta[\tau_1(\tau_1+3) + \tau_2(\tau_2+1)] \\ + \epsilon J(J+1) + \eta L(L+1). \end{aligned} \tag{13}$$

The coefficients α, β, γ, δ, ϵ and η have been determined in a simultaneous fit of the excitation energies of the four nuclei of figure 1 [14].

In a dynamical supersymmetry, closed expressions can be derived for energies, and selection rules and intensities for electromagnetic transitions and single-particle transfer reactions. While a simultaneous description and classification of these observables in terms of the $U(6/12)_\nu \otimes U(6/4)_\pi$ supersymmetry has been shown to be fulfilled to a good approximation for the quartet of nuclei ^{194}Pt, ^{195}Au, ^{195}Pt and ^{196}Au, there are important predictions still not fully verified by experiments. These tests involve the transfer reaction intensities among the supersymmetric partners. In the next section we concentrate on the latter and, in particular, on the one-proton transfer reactions $^{194}\mathrm{Pt} \to {}^{195}\mathrm{Au}$ and $^{195}\mathrm{Pt} \to {}^{196}\mathrm{Au}$.

4. One-proton transfer reactions

The single-particle transfer operator that is commonly used in the IBFM has been derived in the seniority scheme [19]. Although strictly speaking this derivation is only valid in the vibrational regime, it has been used for deformed nuclei as well. An alternative method is based on symmetry considerations. It consists in expressing the single-particle transfer operator in terms of tensor operators under the subgroups that appear in the group chain of a dynamical (super)symmetry [20–22]. The use of tensor operators to describe single-particle transfer

Table 1. Intensities of one-proton transfer reactions for the transfer operator of equation (14). For the supersymmetric quartet of nuclei ^{194,195}Pt–^{195,196}Au the boson numbers are $N_\nu = 4$, $N_\pi = 1$ and $N = N_\nu + N_\pi = 5$, see figure 1.

$^{194}\mathrm{Pt} \rightarrow {}^{195}\mathrm{Au}$	$\vert\langle f \Vert P^\dagger \Vert i\rangle\vert^2$
$\langle (N+\frac{3}{2},\frac{1}{2},\frac{1}{2}),(\frac{1}{2},\frac{1}{2}),\frac{3}{2}\vert$	$[\alpha_0(N+5)\sqrt{5}-\alpha_2(N+1)]^2\,\frac{N_\pi+1}{5(N+3)^2}$
$\langle (N+\frac{1}{2},\frac{1}{2},-\frac{1}{2}),(\frac{1}{2},\frac{1}{2}),\frac{3}{2}\vert$	$[\alpha_0\sqrt{5}+\alpha_2]^2\,\frac{(N+1)(N+5)(N_\pi+1)}{5(N+3)^2}$
$^{195}\mathrm{Pt} \rightarrow {}^{196}\mathrm{Au}$	$\vert\langle f \Vert P^\dagger \Vert i\rangle\vert^2$
$\langle (N+\frac{3}{2},\frac{1}{2},\frac{1}{2}),(\frac{1}{2},\frac{1}{2}),\frac{3}{2},L\vert$	$[\alpha_0(N+5)\sqrt{5}-\alpha_2(N+1)]^2\,\frac{N_\pi+1}{5(N+3)^2}\,\frac{2L+1}{4}$
$\langle (N+\frac{1}{2},\frac{1}{2},-\frac{1}{2}),(\frac{1}{2},\frac{1}{2}),\frac{3}{2},L\vert$	$[\alpha_0\sqrt{5}+\alpha_2]^2\,\frac{(N+1)(N+5)(N_\pi+1)}{5(N+3)^2}\,\frac{2L+1}{4}$

reactions in the supersymmetry scheme has the advantage of giving rise to selection rules and closed expressions for the spectroscopic factors, whose consequences for the experimental observables can be better gauged. The single-particle transfer between different members of the same supermultiplet provides an important test of supersymmetries, since it involves the transformation of a boson into a fermion or vice versa, but conserving the total number of bosons plus fermions.

The one-proton transfer operator in the $U(6/12)_\nu \otimes U(6/4)_\pi$ supersymmetry consists, in lowest order, of two terms

$$P^\dagger = \alpha_0\big(\tilde{s}_\pi \times a^\dagger_{\pi,3/2}\big)^{(3/2)} + \alpha_2\big(\tilde{d}_\pi \times a^\dagger_{\pi,3/2}\big)^{(3/2)} \tag{14}$$

that describe the one-proton transfer reactions between the Pt and Au nuclei belonging to the quartet of nuclei of equation (1): $^{194}\mathrm{Pt} \rightarrow {}^{195}\mathrm{Au}$ and $^{195}\mathrm{Pt} \rightarrow {}^{196}\mathrm{Au}$. In table 1 we present the intensities of the allowed one-proton transfer reactions from the ground state $|(N+2,0,0),(0,0),0\rangle$ of the even–even nucleus ^{194}Pt to the even–odd nucleus ^{195}Au belonging to the same supermultiplet $[\mathcal{N}_\nu\} \otimes [\mathcal{N}_\pi\} = [N_\nu+1\} \otimes [N_\pi+1\}$. The intensity is defined as

$$I = |\langle f \| P^\dagger \| i\rangle|^2. \tag{15}$$

The transfer operator of equation (14) is a tensor operator under Spin(5) and Spin(3). Its transformation properties are $(\tau_1,\tau_2) = (1/2,1/2)$ under Spin(5) and $J = 3/2$ under Spin(3). Due to the selection rules, $P^\dagger$ can only excite states in the final nucleus with $(\tau_1,\tau_2) = (1/2,1/2)$ and $J = 3/2$. The allowed values of the Spin(6) labels are $(\sigma_1,\sigma_2,\sigma_3) = (N+3/2,1/2,1/2)$ for the ground state and $(N+1/2,1/2,-1/2)$ for an excited state. The ratio of the intensities for the excitation of the excited and the ground state is given by

$$R = \frac{I_{\mathrm{gs}\rightarrow\mathrm{exc}}}{I_{\mathrm{gs}\rightarrow\mathrm{gs}}} = (N+1)(N+5)\left[\frac{\alpha_0\sqrt{5}+\alpha_2}{\alpha_0(N+5)\sqrt{5}-\alpha_2(N+1)}\right]^2. \tag{16}$$

The number of bosons N is taken to be the number of bosons in the odd–odd nucleus ^{196}Au: $N = N_\nu + N_\pi = 4+1 = 5$, see figure 1. In the bottom half of table 1, we show the allowed transitions for the one-proton transfer from the ground state $|(N+2,0,0),(0,0),0,1/2\rangle$ of the odd–even nucleus ^{195}Pt to the odd–odd nucleus ^{196}Au. In this case, the transfer operator of equation (14) excites doublets of ^{196}Au characterized by $(\tau_1,\tau_2) = (1/2,1/2)$, $J = 3/2$ and $L = J \pm 1/2$, belonging to the ground band with $(\sigma_1,\sigma_2,\sigma_3) = (N+3/2,1/2,1/2)$, and to an excited band with $(N+1/2,1/2,-1/2)$. The ratio of the intensities is the same as that for the $^{194}\mathrm{Pt} \rightarrow {}^{195}\mathrm{Au}$ transfer reaction in equation (16),

$$R(^{195}\mathrm{Pt} \rightarrow {}^{196}\mathrm{Au}) = R(^{194}\mathrm{Pt} \rightarrow {}^{195}\mathrm{Au}). \tag{17}$$

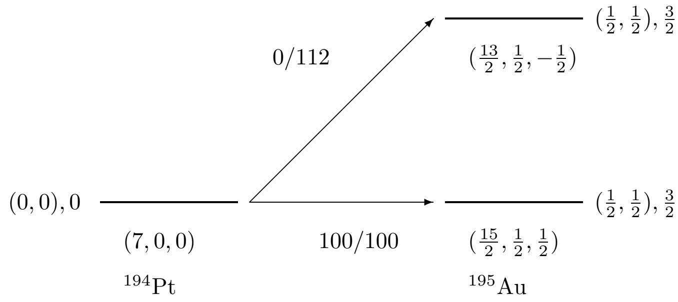

Figure 2. Allowed one-proton transfer reactions for ^{194}Pt $\rightarrow$ ^{195}Au. The spectroscopic factors are normalized to 100 for the ground state to ground state transition for the operators P_1/P_2.

Table 2. As table 1, but for the transfer operators of equation (18).

^{194}Pt $\rightarrow$ ^{195}Au	$\lvert\langle f \Vert P_1^{\dagger} \Vert i\rangle\rvert^2$	$\lvert\langle f \Vert P_2^{\dagger} \Vert i\rangle\rvert^2$
$\langle(N+\frac{3}{2}, \frac{1}{2}, \frac{1}{2}), (\frac{1}{2}, \frac{1}{2}), \frac{3}{2}\rvert$	$\frac{2(N_\pi+1)}{3}\alpha^2$	$\frac{8(N+6)^2(N_\pi+1)}{15(N+3)^2}\alpha^2$
$\langle(N+\frac{1}{2}, \frac{1}{2}, -\frac{1}{2}), (\frac{1}{2}, \frac{1}{2}), \frac{3}{2}\rvert$	0	$\frac{6(N+1)(N+5)(N_\pi+1)}{5(N+3)^2}\alpha^2$
^{195}Pt $\rightarrow$ ^{196}Au	$\lvert\langle f \Vert P_1^{\dagger} \Vert i\rangle\rvert^2$	$\lvert\langle f \Vert P_2^{\dagger} \Vert i\rangle\rvert^2$
$\langle(N+\frac{3}{2}, \frac{1}{2}, \frac{1}{2}), (\frac{1}{2}, \frac{1}{2}), \frac{3}{2}, L\rvert$	$\frac{2(N_\pi+1)}{3}\,\frac{2L+1}{4}\alpha^2$	$\frac{8(N+6)^2(N_\pi+1)}{15(N+3)^2}\,\frac{2L+1}{4}\alpha^2$
$\langle(N+\frac{1}{2}, \frac{1}{2}, -\frac{1}{2}), (\frac{1}{2}, \frac{1}{2}), \frac{3}{2}, L\rvert$	0	$\frac{6(N+1)(N+5)(N_\pi+1)}{5(N+3)^2}\,\frac{2L+1}{4}\alpha^2$

This is a direct consequence of the supersymmetry classification of the states.

For special choices of α_0 and α_2, the transfer operator of equation (14) becomes a tensor operator under Spin(6) as well,

$$\begin{aligned} P_1^{\dagger} &= \alpha\left[-\sqrt{\tfrac{1}{6}}\left(\tilde{s}_\pi \times a^{\dagger}_{\pi,3/2}\right)^{(3/2)} + \sqrt{\tfrac{5}{6}}\left(\tilde{d}_\pi \times a^{\dagger}_{\pi,3/2}\right)^{(3/2)}\right] \\ P_2^{\dagger} &= \alpha\left[+\sqrt{\tfrac{5}{6}}\left(\tilde{s}_\pi \times a^{\dagger}_{\pi,3/2}\right)^{(3/2)} + \sqrt{\tfrac{1}{6}}\left(\tilde{d}_\pi \times a^{\dagger}_{\pi,3/2}\right)^{(3/2)}\right]. \end{aligned} \tag{18}$$

Here $P_1^{\dagger}$ transforms as $(\sigma_1, \sigma_2, \sigma_3) = (1/2, 1/2, -1/2)$ under Spin(6), and $P_2^{\dagger}$ as $(3/2, 1/2, 1/2)$. Due to the Spin(6) selection rules, the operator $P_1^{\dagger}$ only excites the ground state of the Au nuclei with $(\sigma_1, \sigma_2, \sigma_3) = (N + 3/2, 1/2, 1/2)$, whereas $P_2^{\dagger}$ populates, in addition to the ground state, also an excited state with $(N + 1/2, 1/2, -1/2)$. In table 2, we present the intensities of the allowed transfers for the operators of equation (18). These correspond to special cases of the more general results of table 1. Figures 2 and 3 show the allowed transitions for the one-proton transfer reaction ^{194}Pt $\rightarrow$ ^{195}Au and ^{195}Pt $\rightarrow$ ^{196}Au, respectively. The ratio of the intensities is now given by

$$\begin{aligned} R_1(^{195}\mathrm{Pt} \rightarrow {}^{196}\mathrm{Au}) &= R_1(^{194}\mathrm{Pt} \rightarrow {}^{195}\mathrm{Au}) = 0 \\ R_2(^{195}\mathrm{Pt} \rightarrow {}^{196}\mathrm{Au}) &= R_2(^{194}\mathrm{Pt} \rightarrow {}^{195}\mathrm{Au}) = \frac{9(N+1)(N+5)}{4(N+6)^2} \end{aligned} \tag{19}$$

for P_1 and P_2, respectively. For the one-proton transfer reactions ^{194}Pt $\rightarrow$ ^{195}Au and ^{195}Pt $\rightarrow$ ^{196}Au, the second ratio is given by $R_2 = 1.12$ $(N = 5)$.

The available experimental data from the proton stripping reactions ^{194}Pt $(\alpha, \mathrm{t})^{195}$Au and ^{194}Pt $(^3\mathrm{He, d})^{195}$Au [23] show that the $J = 3/2$ ground state of ^{195}Au is excited strongly with

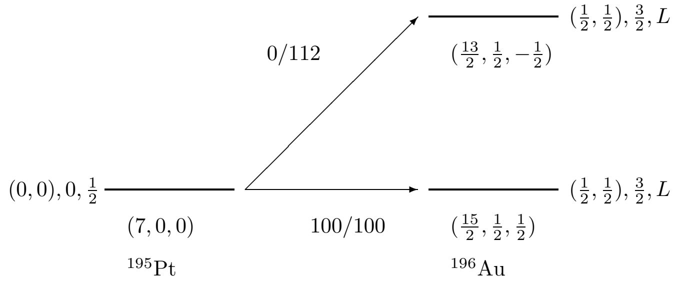

Figure 3. As figure 2, but for $^{195}\mathrm{Pt} \rightarrow {}^{196}\mathrm{Au}$.

$C^2S = 0.175$, whereas the first excited $J = 3/2$ state is excited weakly with $C^2S = 0.019$. In the SUSY scheme, the latter state is assigned as a member of the ground-state band with $(\tau_1, \tau_2) = (5/2, 1/2)$. Therefore the one proton transfer to this state is forbidden by the Spin(5) selection rule of the tensor operators of equation (18). The relatively small strength to excited $J = 3/2$ states suggests that the operator P_1 of equation (18) can be used to describe the data with a good degree of approximation.

According to equations (17) and (19), the ratio of the intensities for the $^{195}\mathrm{Pt} \rightarrow {}^{196}\mathrm{Au}$ transfer reaction is the same as that for $^{194}\mathrm{Pt} \rightarrow {}^{195}\mathrm{Au}$. The equality of the ratios is a consequence of the supersymmetry classification. This prediction will be tested experimentally using the (^{3}He, d) reaction on ^{194}Pt and ^{195}Pt targets [24].

5. Correlations

As we have seen in the previous section, the matrix elements for one-proton transfer reactions between odd-neutron and odd–odd nuclei are related to those between even–even and odd-proton nuclei. The results were obtained by deriving the matrix elements and taking the ratios. However, it is possible to generalize these results and to establish explicit relations between the intensities of these two transfer reactions, i.e. the one-proton transfer reaction intensities between the (ground state of the) Pt and Au nuclei are related by

$$I({}^{195}\mathrm{Pt} \rightarrow {}^{196}\mathrm{Au}) = \frac{2L+1}{4} I({}^{194}\mathrm{Pt} \rightarrow {}^{195}\mathrm{Au}). \tag{20}$$

This correlation holds for both the general form of the transfer operator of equation (14) and the two tensor operators of equation (18). It can be derived from the symmetry relations that exist between the different $U(6)$ couplings in the wavefunctions of the even–even, odd–even, even–odd and odd–odd nuclei of a supersymmetric quartet (the so-called F-spin properties [4]). As a consequence, it is sufficient to derive the intensities for one of the reactions only. The intensities for the other reaction can then be obtained immediately from the correlation in equation (20).

For the one-neutron transfer reactions, $^{194}\mathrm{Pt} \leftrightarrow {}^{195}\mathrm{Pt}$ and $^{195}\mathrm{Au} \leftrightarrow {}^{196}\mathrm{Au}$, there exists a similar situation. We have found some preliminary results for correlations among different reactions which are similar, but not identical, to those obtained for one-proton transfer in equation (20).

There are still two other possible tests that probe directly the structure of the wavefunctions of a supermultiplet of nuclei. (i) The two-nucleon transfer reaction $^{194}\mathrm{Pt}\,(\alpha, \mathrm{d})^{196}\mathrm{Au}$ that has been measured recently [24], in which a neutron–proton pair is transferred to the target nucleus. This reaction presents a very sensitive test of the wavefunctions, since it is not only a measure for the transfer intensity, but it also probes the correlation within the transferred neutron–proton pair. (ii) The charge-exchange reaction ^{195}Au $\rightarrow$ ^{195}Pt (also connected to the β decay which has been studied in the IBFM in [25]). Theoretically, both processes involve a combination of the operator for one-proton and for one-neutron transfer reactions inside the same supermultiplet.

In principle, the available experimental data from the proton stripping reactions $^{194}\mathrm{Pt}(\alpha, \mathrm{t})^{195}\mathrm{Au}$ and $^{194}\mathrm{Pt}(^{3}\mathrm{He}, \mathrm{d})^{195}\mathrm{Au}$ [23] and from the neutron stripping reaction $^{194}\mathrm{Pt}(\mathrm{d}, \mathrm{p})^{195}\mathrm{Pt}$ [26] can be used to determine the appropriate form of the one-proton and one-neutron transfer operators [21], which can then be used to predict the spectroscopic factors for the other one-nucleon transfer reactions between nuclei belonging to the quartet of figure 1, e.g. ^{195}Au $\rightarrow$ ^{196}Au and ^{195}Pt $\rightarrow$ ^{196}Au, as well as for the two-nucleon transfer reaction $^{194}\mathrm{Pt}(\alpha, \mathrm{d})^{196}\mathrm{Au}$ and the log *ft* values of the β decay ^{195}Au $\rightarrow$ ^{195}Pt [27].

6. Summary, conclusions and outlook

The recent measurements of the spectroscopic properties of the odd–odd nucleus ^{196}Au have rekindled interest in nuclear supersymmetry. The available data on the spectroscopy of the quartet of nuclei ^{194}Pt, ^{195}Au, ^{195}Pt and ^{196}Au can, to a good approximation, be described in terms of the $U(6/4)_\pi \otimes U(6/12)_\nu$ supersymmetry. However, there is a still an important set of experiments which can further test the predictions of the supersymmetry scheme: transfer reactions between nuclei belonging to the same supermultiplet, in particular between the odd–even (and even–odd) and odd–odd members of the supersymmetric quartet. Theoretically, these transfers are described by the supersymmetric generators which change a boson into a fermion, or vice versa. Most available data involve transfer reactions between nuclei belonging to different multiplets.

In this paper, we investigated one-proton transfer reactions between the SUSY partners: ^{194}Pt$\rightarrow$ ^{195}Au and ^{195}Pt$\rightarrow$ ^{196}Au. The supersymmetry implies an explicit correlation between the spectroscopic factors of these two reactions which can be tested experimentally. Preliminary results suggest that for the one-neutron transfer reactions ^{194}Pt $\leftrightarrow$ ^{195}Pt and ^{195}Au $\leftrightarrow$ ^{196}Au there exist correlations similar to those obtained for the one-proton transfer in equation (20). To the best of our knowledge, this is the first time that such relations have been predicted for nuclear reactions among different pairs of nuclei, which may provide a challenge and motivation for future experiments.

An extension of these ideas can be applied to the two-nucleon transfer reaction ^{194}Pt $(\alpha, \mathrm{d})^{196}$Au and the charge-exchange reaction (or β decay) ^{195}Au $\rightarrow$ ^{195}Pt. Even though they may represent different physical processes, i.e. one- and two-nucleon transfer reactions and β decay, the nuclear structure contributions are related by supersymmetry. Whether it is possible to find a simultaneous description in which all of these processes are correlated by SUSY is an open question [27].

In this paper, we have discussed n-SUSY in combination with dynamical symmetries. However, dynamical symmetries are very scarce and have severely limited the study of nuclear supersymmetry. An example of n-SUSY without dynamical symmetry is a study of the Ru and Rh isotopes in the $U(6/12)$ supersymmetry, in which a combination of the $U^{BF}(5)$ and $SO^{BF}(6)$ dynamical symmetries was shown to give an excellent description of the data [3]. This opens up the possibility of generalizing n-SUSY to transitional regions of the nuclear mass table, to find other examples of supersymmetric quartets of nuclei, and to further extend the search for correlations as a result of SUSY. Of course, it remains to be seen whether the correlations predicted by n-SUSY are verified by future experiments and whether these correlations can be truly extended to other regions of the nuclear table. If this is indeed the case, nuclear supersymmetry may yet provide a powerful unifying scheme for atomic nuclei.

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